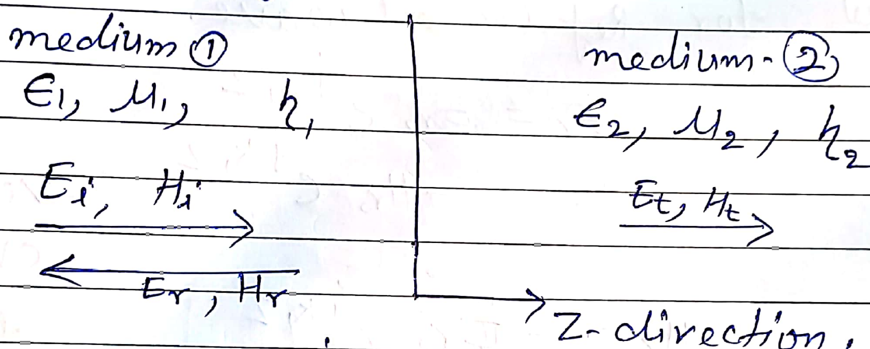
Refraction and Reflection of electromagnetic waves.

Reflection of e.m wave at normal incidence:-

Let us consider two medium whose permittivity and permeability are ϵ_1, μ_1 and ϵ_2, μ_2.



where $E_i = E_{i_0} e^{-jk_1 z}$ $\qquad$ k_1 is the propagation vector for incident wave.

$H_i = H_{i_0} e^{-jk_1 z}$

where η_1 and η_2 are the intrinsic impedance then

we know that $\eta = \frac{E}{H} \Rightarrow H = \frac{E}{\eta}$

$$H_i = \frac{E_{i0}}{\eta_1} e^{-jk_1 z} \quad \text{---} (1)$$

For transmitted wave.

$$E_t = E_{t_0} e^{-jk_2 z}$$

$$H_t = H_{t_0} e^{-jk_2 z}$$

$$H_t = \frac{E_{t_0}}{\eta_2} e^{-jk_2 z} \quad \text{---} (2)$$

$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$, $n = \sqrt{\mu_r \epsilon_r}$ $\Rightarrow \eta = \frac{\mu_0}{\sqrt{\mu_0 \epsilon_0}} = \frac{\mu_0}{n}$ $\quad \boxed{\eta = \frac{1}{n}}$ $\quad \mu_0 = 1$

From boundary condition

$$H_{tan-1} = H_{tan-2}$$

$$H_i = H_t$$

$$\frac{E_{io}}{\eta_1} e^{-jk_1 z} = \frac{E_{to}}{\eta_2} e^{-jk_2 z} \quad \text{--- (2)}$$

Now for Reflected wave.

$$E_r = E_{ro} e^{jk_1 z}$$

$$H_r = -H_{ro} e^{jk_1 z}$$

(-ve sign is due to change in the direction of magnetic field.)

$$H_r = -\frac{E_{ro}}{\eta_1} e^{jk_1 z}$$

at $z = 0$

$$E_i + E_r = E_t \quad \text{--- (A)}$$

$$H_i + H_r = H_t \quad \text{--- (B)}$$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$\frac{E_i - E_r}{\eta_1} = \frac{E_i + E_r}{\eta_2} \quad \text{from (A)}$$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_i}{\eta_2} + \frac{E_r}{\eta_2}$$

$$\frac{E_i}{\eta_1} - \frac{E_i}{\eta_2} = \frac{E_r}{\eta_1} + \frac{E_r}{\eta_2}$$

$$E_i\left(\frac{1}{\eta_1}-\frac{1}{\eta_2}\right)=E_r\left(\frac{1}{\eta_1}+\frac{1}{\eta_2}\right)$$

$$E_i\left(\frac{\eta_2-\eta_1}{\cancel{\eta_1\eta_2}}\right)=E_r\left(\frac{\eta_2+\eta_1}{\cancel{\eta_1\eta_2}}\right)$$

$$\frac{\eta_2-\eta_1}{\eta_2+\eta_1}=\frac{E_r}{E_i}$$

$$\frac{E_r}{E_i}=\left(\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right)$$

Reflection coefficient (R) is define as

$$R=\left|\frac{E_r}{E_i}\right|^2$$

$$\boxed{R=\left(\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right)^2}$$

Now $$\frac{E_i}{\eta_1}-\frac{E_r}{\eta_1}=\frac{E_t}{\eta_2}$$

$$\frac{E_i}{\eta_1}-\frac{(E_t-E_i)}{\eta_1}=\frac{E_t}{\eta_2}$$

$$\frac{E_i}{\eta_1}+\frac{E_i}{\eta_1}-\frac{E_t}{\eta_1}=\frac{E_t}{\eta_2}$$

$$E_i\left(\frac{1}{\eta_1}+\frac{1}{\eta_1}\right)=E_t\left(\frac{1}{\eta_1}+\frac{1}{\eta_2}\right)$$

$$E_i \left(\frac{\eta_1 + \eta_1}{\eta_1 \cancel{\eta_1}} \right) = E_t \left(\frac{\eta_2 + \eta_1}{\cancel{\eta_1} \eta_2} \right)$$

$$\frac{2\eta_2}{\eta_1 + \eta_2} = \frac{E_t}{E_i}$$

$$\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Transmission coefficient (T) is define as

$$T = \frac{\eta_1}{\eta_2} \left| \frac{E_t}{E_i} \right|^2 \Rightarrow \frac{\eta_1}{\cancel{\eta_2}} \times \frac{4\eta_2^{\cancel{2}}}{(\eta_2 + \eta_1)^2}$$

$$\boxed{T = \frac{4\eta_1 \eta_2}{(\eta_2 + \eta_1)^2}}$$